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Magnetopolarons in quasi-one-dimensional quantum-well wires

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Abstract. The interaction of quasi-one-dimensional electrons and longitudinal optical (LO) phonons placed in a perpendicular magnetic field is calculated. Results are presented for the polaron correction to the Landau levels (N) and the polaron cyclotron mass. It is shown that under the condition $\omega_L > N\Omega$ (where ω_L is the LO phonon frequency, and Ω is the frequency of the parabolic confinement potential) level crossing occurs, resulting in a resonant magnetopolaron in the vicinity of the cyclotron frequency $\omega_C = [(\omega_L/N)^2 - \Omega^2]^{1/2}$. The polaron cyclotron mass increases in the vicinity of this ω_C. It is shown that in a cyclotron resonance experiment it is only possible to measure the polaron cyclotron mass if the cyclotron frequency is larger than a critical frequency, determined by the confinement potential.

1. Introduction

The polaron mass is usually determined by a cyclotron resonance experiment. In such an experiment the separation of adjacent Landau levels is measured as a function of the magnetic field B. In polar semiconductors the Landau levels are modified by polaronic effects. Hence, in polar semiconductors the cyclotron resonance frequency $\omega_C^* = eB/m_C^*$, with m_C^* the polaron cyclotron mass, is affected by the interaction of the electrons with the optical phonons. Two situations are commonly distinguished in three-dimensional (3D) and quasi-two-dimensional (Q2D) systems: the non-resonant magnetopolaron in low magnetic fields and the resonant magnetopolaron in quantizing magnetic fields when the cyclotron energy approximately equals the optical phonon energy. For the 3D and Q2D polaron, considerable work has been devoted to the study of the magnetic field dependence of the electron–phonon correction to the energy of Landau levels [1–6].

Advances in epitaxial growth and in nanometre-scale lithography have made it possible to fabricate semiconductor nanostructures which exhibit Q1D and Q0D properties. Q1D quantum-well wires (QWW) and Q0D quantum dots (QD) are produced from InSb metal-oxide semiconductor (MOS) structures and GaAs–$Ga_{1-x}Al_xAs$ heterostructures [7, 8].

In this paper we investigate magnetopolarons in QWW. The electron–phonon correction will be calculated within second-order perturbation theory for arbitrary magnetic fields. Numerical results will be presented for QWW which are produced from InSb MOS structures.

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2. Theory

Using the effective mass approximation, the unperturbed system, a single electron in the presence of a quantizing perpendicular magnetic field is described by the Hamiltonian

$$H_e = \frac{1}{2m_e}(\boldsymbol{p} + e\boldsymbol{A})^2 + V(\boldsymbol{x}) \tag{1}$$

where we neglect the Zeeman spin-splitting. Assuming the Landau gauge $\boldsymbol{A} = (-yB, 0, 0)$ and a parabolic confinement potential in the y direction, $V(\boldsymbol{x}) = m_e\Omega^2 y^2/2 + V(z)$, we have

$$H_e = \frac{\boldsymbol{p}^2}{2m_e} - \omega_C y p_x + \tfrac{1}{2} m_e \tilde{\omega}_C^2 y^2 + V(z) \tag{2}$$

with the cyclotron frequency $\omega_C = eB/m_e$ and the hybrid frequency $\tilde{\omega}_C = (\omega_C^2 + \Omega^2)^{1/2}$. Making the ansatz

$$\langle \boldsymbol{x} | N, k_x \rangle = \Psi_{Nk_x}(\boldsymbol{x}) = \frac{1}{\sqrt{L_x}} e^{ik_x x} \Phi_N(y) \varphi(z) \tag{3}$$

for the single-particle wavefunction, where L_x is the length of the sample in the x direction, the Schrödinger equation for the electron motion in the y direction is

$$\left(\frac{\hbar^2 k_x^2}{2\tilde{m}} - \frac{\hbar^2}{2m_e} \frac{d^2}{dy^2} + \tfrac{1}{2} m_e \tilde{\omega}_C^2 (y - Y)^2 \right) \Phi_N(y) = \mathcal{E}_N(k_x) \Phi_N(y) \tag{4}$$

with $\tilde{m} = m_e(\tilde{\omega}_C/\Omega)^2$ the renormalized mass. We assume that the electron is confined in a zero-thickness x–y plane along the z direction at $z = 0$. Hence, $|\varphi(z)|^2 = \delta(z)$ is valid. The solution of (4) is a shifted harmonic-oscillator wavefunction

$$\Phi_N(y - Y) = \frac{1}{\sqrt{2^N N! \pi^{1/2} \tilde{l}_0}} \exp\left(-\frac{1}{2\tilde{l}_0^2} (y - Y)^2 \right) H_N\left(\frac{1}{\tilde{l}_0} (y - Y) \right) \tag{5}$$

with the centre coordinate $Y = \gamma \tilde{l}_0^2 k_x$, $\tilde{l}_0^2 = \hbar/(m_e \tilde{\omega}_C)$ is the typical width of the wavefunction and $\gamma = \omega_C/\tilde{\omega}_C$; $H_N(\xi)$ is the Hermite polynomial. The corresponding eigenenergies are

$$\mathcal{E}_N(k_x) = \hbar\tilde{\omega}_C (N + \tfrac{1}{2}) + \frac{\hbar^2 k_x^2}{2\tilde{m}} \qquad N = 0, 1, 2, \ldots. \tag{6}$$

Compared with the three- and two-dimensional case in a perpendicular magnetic field, the degeneracy of the Landau levels has been broken by the confinement potential in the y direction. In a classical picture, the term $\hbar^2 k_x^2/2\tilde{m}$ arises from the electrons skip along the two edges of the wire due to the lateral confinement potential and the Lorentz force.

Our interest is directed to QWW created via field effect from a heterostructure [7, 8]. Hence, the optical phonons interacting with the electrons are those of the original layered semiconductor structure. Neglecting the effects of interface phonons [9, 10], the Hamiltonian of the electron–phonon interaction H_{ep} including only 3D bulk longitudinal optical (LO) phonons is the standard Fröhlich Hamiltonian [11]. It is

$$H_{ep} = \left(\frac{4\pi\alpha r_p (\hbar\omega_L)^2}{V_G} \right)^{1/2} \sum_{\boldsymbol{q}} e^{i\boldsymbol{q}\boldsymbol{x}} \frac{1}{|\boldsymbol{q}|} (a_L(\boldsymbol{q}) + a_L^+(-\boldsymbol{q})) \tag{7}$$

where

$$\alpha = \frac{1}{2}\frac{e^2}{4\pi\varepsilon_0 r_p}\left(\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_s}\right)\frac{1}{\hbar\omega_L}$$

the dimensionless 3D polaron coupling constant, $r_p = (\hbar/2m_e\omega_L)^{1/2}$ the corresponding 3D polaron radius, ω_L the frequency of the LO phonons and ε_∞ and ε_s are the high frequency (optical) and the static dielectric constant of the semiconductor containing the Q0D confined electrons, respectively; $a_L(q)$ and $a_L^+(q)$ are the phonon destruction and creation operators, respectively, $q = (q_x, q_y, q_z)$ is the 3D wavevector of the 3D bulk LO phonon and V_G is the volume of the sample. The energy levels of an electron are shifted about $\Delta E_N(k_x)$ by the interaction with the LO phonons:

$$E_N(k_x) = \hbar\tilde{\omega}_C(N + \tfrac{1}{2}) + \frac{\hbar^2 k_x^2}{2\tilde{m}} + \Delta E_N(k_x). \tag{8}$$

Therefore, a phonon continuum with threshold energy $E_{th} = \hbar\omega_L + \hbar\tilde{\omega}_C/2 + \Delta E_0$ exists. Within second-order perturbation theory the energy shift of the Nth Landau level is given by

$$\Delta E_N(k_x) = -\sum_{N'=0}^{\infty}\sum_{q}\frac{|M_{N'N}(q)|^2}{D_{N'N}} \tag{9}$$

where the matrix element is $M_{N'N}(q) = \langle N', k_x - q_x; 1_q|H_{ep}|N, k_x; 0_q\rangle$. The ket $|N, k_x; n_q\rangle = |N, k_x\rangle \otimes |n_q\rangle$ describes a state composed of an electron in the Landau level N with momentum $\hbar k_x$ and n LO phonons with momentum $\hbar q$ and energy $\hbar\omega_L$. Because we only consider weakly polar semiconductors with $\alpha \ll 1$, i.e. the weak-coupling limit, it is sufficient to consider perturbed states containing not more than one LO phonon. The energy dominator in (9) is given by

$$D_{N'N} = \hbar\omega_L + \hbar\tilde{\omega}_C(N' - N) + \frac{\hbar^2 q_x^2}{2\tilde{m}} - \frac{\hbar^2}{\tilde{m}}k_x q_x - \Delta_N \tag{10}$$

where the value of Δ_N depends on the type of the perturbation theory used [2]: (i) $\Delta_N = 0$ leads to Rayleigh–Schrödinger perturbation theory (RSPT), (ii) $\Delta_N = \Delta E_N$ results in the Wigner–Brillouin perturbation theory (WBPT) and (iii) $\Delta_N = \Delta E_N - \Delta E_0^{RSPT}$ gives an improved Wigner–Brillouin perturbation theory (IWBPT), with ΔE_0^{RSPT} is the weak-coupling electron–phonon correction to the electron ground-state energy calculated within RSPT. For the ground-state $\Delta E_0^{IWBPT} = \Delta E_0^{RSPT}$ is valid. Introducing in (9) 3D polaron units (energies are measured in units of $\hbar\omega_L$ and lengths are in units of r_p) and converting the sum over the phonon momentum into an integral, one gets

$$\Delta E_N(k_x) = -\frac{\alpha}{2\pi}\int_0^\infty dq_\parallel \int_{-\pi}^{\pi} d\varphi\, e^{-a}\sum_{N'=0}^{\infty}\frac{N_2!}{N_1!}a^{N_1-N_2} \times \frac{|L_{N_2}^{N_1-N_2}(a)|^2}{1 + \lambda^2(N' - N) + q_\parallel^2\cos^2\varphi(1-\gamma^2) - 2k_x q_\parallel\cos\varphi(1-\gamma^2) - \Delta_N} \tag{11}$$

with

$$a = \frac{q_\parallel^2}{\lambda^2}[1 - (1-\gamma^2)\cos^2\varphi]$$

and $N_1 = \max(N, N')$ and $N_2 = \min(N, N')$, $q_\parallel = |q_\parallel| = (q_x^2 + q_y^2)^{1/2}$, $\lambda^2 = \tilde{\omega}_C/\omega_L$ and where we have introduced cylindrical coordinates in the $q_x - q_y$ plane; $L_N^{N'}(\xi)$ is the associated Laguerre polynomial. The RSPT describes the ground-state correction for $\omega_C \to 0$ quite well, but it fails for the excited states, since it is possible that the denominator vanishes for certain ω_C. This is true because the energy level $(N, k_x = 0; n_q = 0_q)$ of the state $|N, 0; 0_q\rangle$ crosses under the condition $\omega_L > N\Omega$ the energy level $(0, 0; 1_q)$ of the state $|0, 0; 1_q\rangle$ at $\omega_C = [(\omega_L/N)^2 - \Omega^2]^{1/2}$. Under the condition $\omega_L \leqslant N\Omega$ there is no level crossing. Hence, unlike the 3D and Q2D systems where the corresponding level crossing occurs at $N\omega_C = \omega_L$, independent of the geometry of the quantum well, for the Q1D *parabolic confinement the occurrence of the resonance strongly depends on the confinement* energy $\hbar\Omega$. If resonance occurs, the electron–phonon interaction leads to a splitting of the degenerated levels and a pinning to the energy $\hbar\omega_L + \hbar\tilde{\omega}_C/2 + \Delta E_0^{\mathrm{RSPT}}$. The higher energetic branch is within the phonon continuum and is not calculated in this paper. Only the IWBPT gives the correct pinning behaviour in the weak-coupling limit.

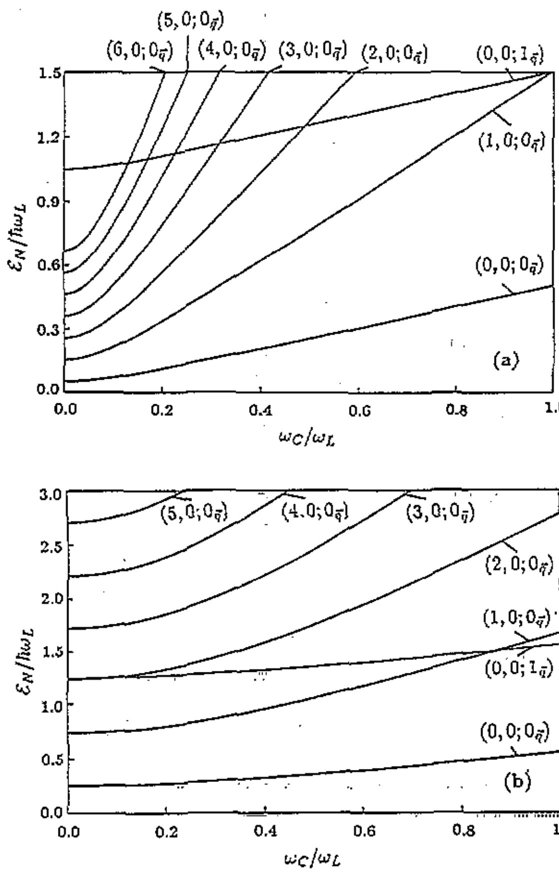

Figure 1. The first unperturbed energy levels $(N, 0; n_q)$ as a function of the magnetic field in an InSb QWW (thick full curve). The thin curve corresponds to the unperturbed level $(0, 0; 1_q)$. In figure 1(*a*) the energy levels are plotted for $\hbar\Omega = 2.5$ meV and in figure 1(*b*) for $\hbar\Omega = 12$ meV where $\hbar\omega_L = 24.41$ meV.

In figure 1 the unperturbed energy levels for one electron in an InSb QWW are plotted as a function of the magnetic field for two different confinement potentials: (*a*) $\hbar\Omega = 2.5$ meV and (*b*) $\hbar\Omega = 12$ meV. The energy levels change their character from subband-like levels for $B \to 0$: $\mathcal{E}_N(k_x)|_{B=0} = \hbar\Omega(N + 1/2) + \hbar^2 k_x^2/2m_e$ to Landau-like levels for $B \to \infty$: $\mathcal{E}_N(k_x)|_{B\to\infty} \to \hbar\omega_C(N + 1/2)$. Therefore, for $B \to \infty$ each Landau level is degenerate (without spin degeneracy) according to the degeneracy factor $N_L = (eB/h)A$, with $A = L_x L_y$ the area of the x–y plane. The resonance frequencies of the energy levels $(N, 0; 0_q)$ with the energy level $(0, 0; 1_q)$ are at $\omega_C = [(\omega_L/N)^2 - \Omega^2]^{1/2}$. Figure 1(*b*) shows that the resonance frequency of the lowest excited level $(1, 0; 0_q)$ with the level $(0, 0; 1_q)$ at $\omega_C = (\omega_L^2 - \Omega^2)^{1/2}$ is shifted to much lower values than the corresponding resonance frequency $\omega_C = \omega_L$ of 3D or Q2D systems if the confinement frequency Ω is increased.

The polaron hybrid frequency $\hbar\tilde{\omega}_C^* = E_N - E_{N-1}$, defines a polaron cyclotron mass $m_C^* = e\hbar B[(E_N - E_{N-1})^2 - (\hbar\Omega)^2]^{-1/2}$. In experiment the optical transition $E_0 \to E_1$ is mostly used, so that it is common to define the mass for this transition:

$$\frac{m_C^*}{m_e} = \left[1 + \left(\frac{\Delta E_1 - \Delta E_0}{\gamma\lambda^2}\right)^2 + 2\left(\frac{\Delta E_1 - \Delta E_0}{\gamma^2\lambda^2}\right)\right]^{-1/2}. \tag{12}$$

Taking the limit of vanishing confinement frequency ($\Omega \to 0$; $\gamma \to 1$; $\lambda^2 \to \omega_C/\omega_L$) we get the well known 2D polaron cyclotron mass: $m_C^* = e\hbar B/(E_1 - E_0)$. The limit of vanishing electron–phonon interaction ($\alpha \to 0$) yield for the Q1D polaron cyclotron mass the conduction band-edge mass $m_C^*/m_e = 1$.

To analyse the polaron cyclotron mass we have to calculate ΔE_0 and ΔE_1. For ΔE_0 it is possible to perform the sum over the Landau levels exactly by converting the denominator of (11) into an integral. After some simple analytic calculations we obtain from (11)

$$\begin{aligned}\Delta E_0(k_x) = -\alpha\frac{\lambda}{\sqrt{\pi}}\int_0^\infty \mathrm{d}t\,\frac{\exp[-(1-\Delta_0)t]}{\sqrt{[\exp(-\lambda^2 t) - 1 + \lambda^2 t](1-\gamma^2)}}\\ \times\int_0^{\pi/2}\mathrm{d}\varphi\,\exp\left(\frac{t^2(1-\gamma^2)^2\lambda^2 k_x^2\cos^2\varphi}{1-\exp(-\lambda^2 t) + [\exp(-\lambda^2 t) - 1 + \lambda^2 t](1-\gamma^2)\cos^2\varphi}\right)\\ \times[\cos^2\varphi + B(t)]^{-1/2}\end{aligned} \tag{13}$$

with

$$B(t) = \frac{1 - \exp(-\lambda^2 t)}{[\exp(-\lambda^2 t) - 1 + \lambda^2 t](1-\gamma^2)}.$$

This expression can be expanded in powers of k_x^2: $\Delta E_0 = \Delta E_0(0) + \Delta E_0'(0)k_x^2 + \ldots$, and so the magnetopolaron effective mass $\tilde{m}^*$ for motion in the x direction is $\tilde{m}^*/m_e = (1 + 2\tilde{m}\Delta E_0'(0)/\hbar)^{-1}$. The magnetopolaron ground-state correction is given to order $(k_x^2)^0$ by

$$\Delta E_0(0) = -\alpha\frac{\lambda}{\sqrt{\pi}}\int_0^\infty \mathrm{d}t\,\frac{\exp[-(1-\Delta_0)t]}{\sqrt{\gamma^2[1-\exp(-\lambda^2 t)] + (1-\gamma^2)\lambda^2 t}}\,K\left(\frac{1}{\sqrt{1+B(t)}}\right) \tag{14}$$

where $K(\xi)$ is the complete elliptical integral of the first kind.

Now we consider the energy shift of the Landau level $N = 1$ for $k_x = 0$, which is necessary for the calculation of the polaron cyclotron mass m_C^*. Summarizing over all

Landau levels N' in (11), $\Delta E_1(0)$ is again given by an one-dimensional integral

$$\Delta E_1(0) = -\alpha \frac{\lambda}{\sqrt{\pi}} \int_0^\infty \mathrm{d}t \, \frac{\exp[-(1-\Delta_1)t]}{\sqrt{\gamma^2[1-\exp(-\lambda^2 t)] + (1-\gamma^2)\lambda^2 t}} \times \left\{ K\left(\frac{1}{\sqrt{1+B(t)}}\right) + \frac{2\sinh^2(\frac{1}{2}\lambda^2 t)}{\gamma^2[1-\exp(-\lambda^2 t)] + (1-\gamma^2)\lambda^2 t} \times \left[\gamma^2 K\left(\frac{1}{\sqrt{1+B(t)}}\right) + \lambda^2 t \left(\frac{1-\gamma^2}{(\mathrm{e}^{-\lambda^2 t} - 1 + \lambda^2 t)[\gamma^2(1-\mathrm{e}^{-\lambda^2 t}) + (1-\gamma^2)\lambda^2 t]} \right)^{1/2} \times K'\left(\frac{1}{\sqrt{1+B(t)}}\right) \right] \right\} \tag{15}$$

where $K'(\xi)$ is the first derivative of the complete elliptical integral of the first kind. Thus both interesting energy corrections are given by one-dimensional integrals.

3. Numerical results

For numerical calculation we have used an InSb MOS structure (InSb: $\alpha = 0.0196$, $r_\mathrm{p} = 10.594\,\mathrm{nm}$, $\hbar\omega_\mathrm{L} = 24.41\,\mathrm{meV}$, $m_\mathrm{e} = 0.0139 m_0$) in which the electrons are confined within InSb and a nanostructured GaAs–$\mathrm{Ga_{1-x}Al_xAs}$ heterostructure (GaAs: $\alpha = 0.07$, $r_\mathrm{p} = 3.987\,\mathrm{nm}$, $\hbar\omega_\mathrm{L} = 36.17\,\mathrm{meV}$, $m_\mathrm{e} = 0.06624 m_0$) where the electrons are confined within GaAs. Because in the experimentally realized structures typically $\hbar\Omega \leqslant 12\,\mathrm{meV}$ for InSb and $\hbar\Omega \leqslant 5\,\mathrm{meV}$ for GaAs is valid, we always have the possibility of a resonance at $\omega_\mathrm{C} = (\omega_\mathrm{L}^2 - \Omega^2)^{1/2}$.

The calculated energy levels for one electron in different InSb QWW including polaron effects are plotted in figure 2. The thin full curves show the unperturbed levels $(N, 0; 0_q)$, the thin broken curve the unperturbed level $(0, 0; 1_q)$ and the heavy full curves are the corresponding perturbed levels. The perturbed levels are obtained from (8), (14) and (15), respectively. From figure 2 it is apparent that the perturbed levels, the magnetopolaron levels, are shifted to lower energies $\sim \Delta E_N(\omega_\mathrm{C} = 0)$ independent of the magnetic field, and with increasing magnetic field the state $|1, 0; 0_q\rangle$ mixes strongly with $|0, 0; 1_q\rangle$, becoming resonant near the unperturbed level crossing at $\omega_\mathrm{C} = (\omega_\mathrm{L}^2 - \Omega^2)^{1/2}$. The energy levels are repelled from the level $(0, 0; 1_q)$ and pinned to the energy $\hbar\omega_\mathrm{L} + 1/2\hbar\tilde{\omega}_\mathrm{C} + \Delta E_0^\mathrm{RSPT}$ in the following manner. The levels for which at $B = 0$ for the energy $\mathcal{E}_N = \hbar\Omega(N + 1/2) < \hbar(1/2\Omega + \omega_\mathrm{L})$ is valid, are pinned to this level from the lower energy side. Comparing figures 2(*a*) and 2(*b*) one can see that with increasing Ω of the confinement potential the mixing of the levels $(0, 0; 1_q)$ and $(1, 0; 0_q)$ becomes stronger for smaller magnetic fields.

In cyclotron resonance experiments the transition of electrons between the energy levels is observed. Hence, the transition is detected between the plotted magnetopolaron levels (perturbed or renormalized levels) of figure 2. The energy difference between the two successive Landau levels $E_1 - E_0 = \hbar\tilde{\omega}_\mathrm{C} + \Delta E_1 - \Delta E_0$ is plotted in figure 3 as a function of the magnetic field for different InSb and GaAs–$\mathrm{Ga_{1-x}Al_xAs}$ QWW. The heavy full curves represent the renormalized energy difference and the thin full curves those of the unperturbed levels. Because $\Delta E_1 - \Delta E_0 < 0$ at $B = 0$, the energy difference is a little bit smaller between the renormalized levels than that between the unperturbed levels.

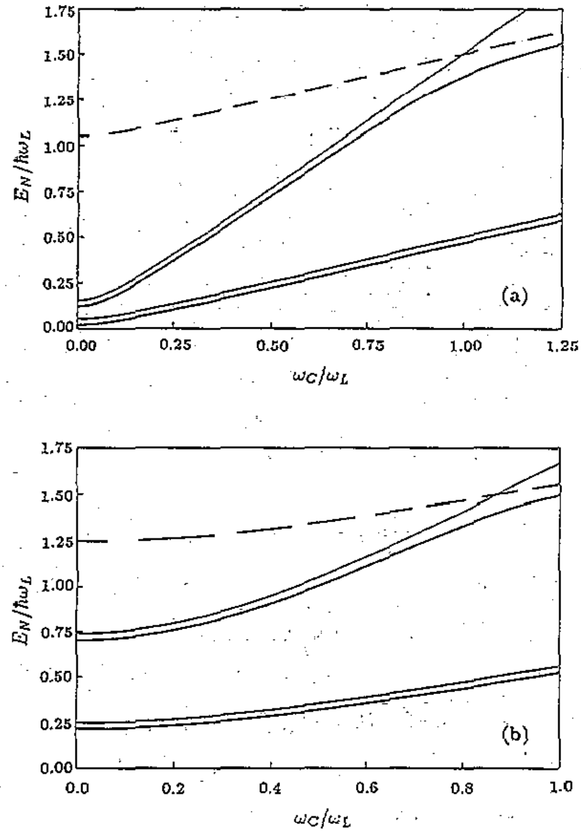

Figure 2. The magnetopolaron levels E_0, E_1 (thick full curves) as a function of the magnetic field in an InSb QWW for $\hbar\Omega = 2.5$ meV (*a*) and $\hbar\Omega = 12$ meV (*b*). The corresponding unperturbed energy levels are plotted by thin full $(0, 0; 0_q)$, $(1, 0; 0_q)$ and broken $(0, 0; 1_q)$ curves.

The energy difference $E_1 - E_0$ becomes equal to the LO phonon energy $\hbar\omega_L$ in the limit $B \to \infty$. One can see that for the GaAs–$Ga_{1-x}Al_xAs$ QWW (figure 3(*b*)) the polaronic effects are more pronounced than for the InSb QWW (figure 3(*a*)). This is visible for both the decreasing of the transition energy in the low magnetic field region and in the resonance region $\omega_C \approx \sqrt{\omega_L^2 - \Omega^2}$.

The Q1D polaron cyclotron mass is shown in figure 4. For small magnetic fields ($B \to 0$) this mass increases with decreasing magnetic fields. The physical reason for this behaviour is that, for $B \to 0$, the confinement in the y direction of the structure increasingly results from the sample geometry and, hence, the quantum size effects causes the particle localization. This novel effect is absent for 3D and Q2D systems in perpendicular magnetic fields. In these systems the polaron cyclotron mass is only slightly higher than the polaron mass for small but finite magnetic fields, but approaching the polaron mass for $B \to 0$. The increase of the mass with increasing magnetic field beginning from the minimum is the polaron-induced non-parabolicity in the absence of the band non-parabolicity. The strong enhancement of the polaron cyclotron mass around $\omega_C = \sqrt{\omega_L^2 - \Omega^2}$ is a consequence of the pinning of the Landau level to the energy $\hbar\omega_L + \hbar\tilde{\omega}_C/2 + \Delta E_0^{RSPT}$. If one compares the results for RSPT, WBPT and IWBPT, figure 4(*a*), we can conclude that for Q1D systems the

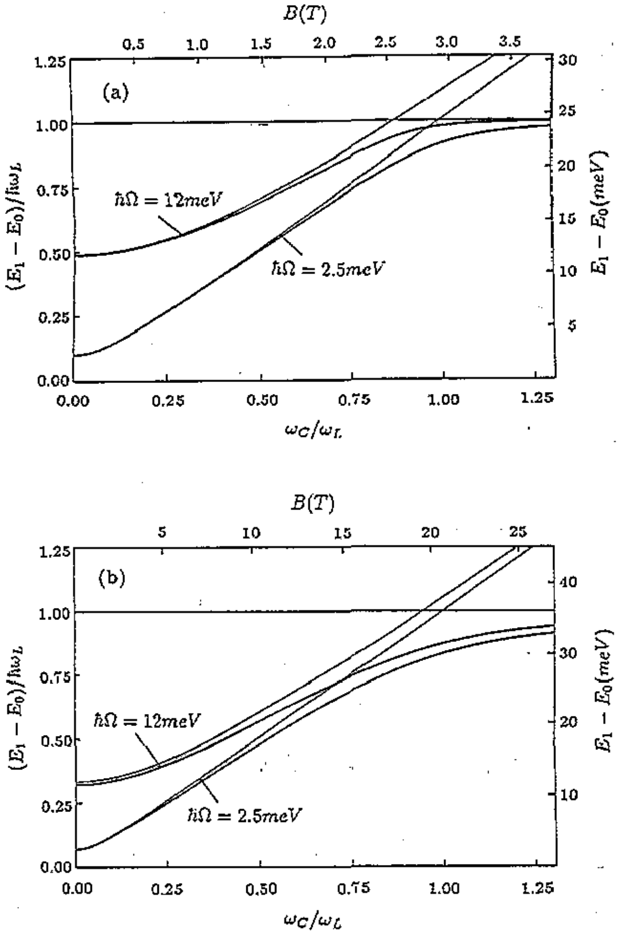

Figure 3. Energy difference between the energy levels $E_1 - E_0$ for the perturbed states (magnetopolaron, thick full curves) and the unperturbed states (electron, thin full curves) for (*a*) an InSb QWW and (*b*) a GaAs–$Ga_{1-x}Al_xAs$ QWW with $\hbar\Omega = 2.5$ meV and $\hbar\Omega = 12$ meV.

same differences between the different types of perturbation theories are valid as for 3D and Q2D systems [4]: RSPT overestimates the contribution of the polaron effects to the polaron mass, WBPT underestimates the polaron effects whereas IWBPT is a good improvement on WBPT.

In figure 4(*b*) the polaron cyclotron mass is plotted against magnetic field for Q1D magnetopolarons with different confinement energies $\hbar\Omega$ and for the strict 2D magnetopolaron which is the limiting case of $\hbar\Omega = 0$ meV. This figure clearly shows the enhancement of the mass renormalization due to polaron effects in systems with reduced dimensionality. It is well known [5] that the polaron cyclotron mass of the 2D magnetopolaron is larger than that of the 3D magnetopolaron. Here we have obtained the following results: (i) the polaron cyclotron mass of the Q1D magnetopolaron is larger than this of the 2D magnetopolaron and (ii) the polaron cyclotron mass increases with increasing confinement energy.

To understand the increase of the Q1D polaron cyclotron mass with decreasing magnetic fields in the low magnetic field region we look at the denominator of (12). This denominator

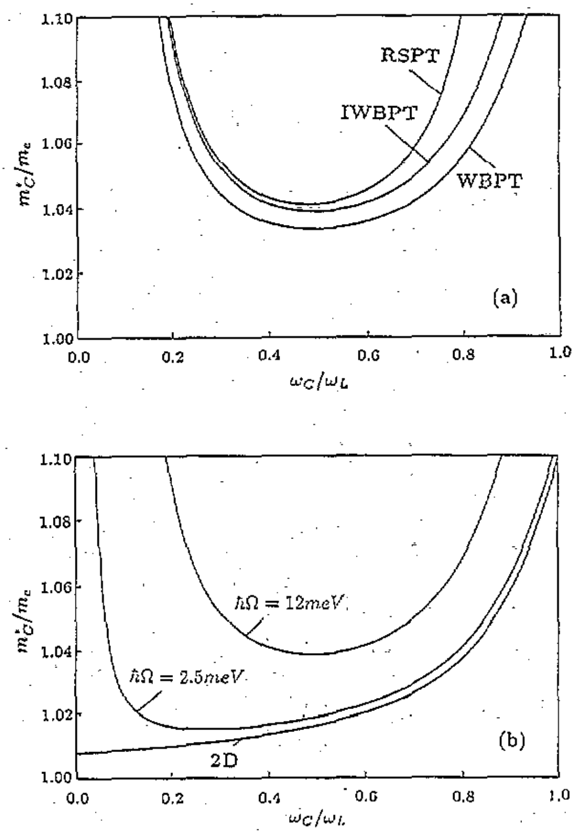

Figure 4. Polaron cyclotron mass of the QID magnetopolaron against magnetic field of an InSb QWW for (*a*) different types of perturbation theories RSPT, WBPT and IWBPT for $\hbar\Omega = 12\,\text{meV}$ and (*b*) different confinement energies $\hbar\Omega = 12\,\text{meV}$ and $\hbar\Omega = 2.5\,\text{meV}$ calculated using IWBPT. The curve, denoted by 2D, is the corresponding cyclotron mass of a 2D magnetopolaron ($\hbar\Omega = 0\,\text{meV}$).

has a zero at

$$\omega_C^c/\omega_L = \{[\Delta E_1(\omega_C^c, \Omega) - \Delta E_0(\omega_C^c, \Omega)]^2 + 2\Omega/\omega_L[\Delta E_1(\omega_C^c, \Omega) - \Delta E_0(\omega_C^c, \Omega)]\}^{1/2}. \quad (16)$$

This zero, called the critical cyclotron frequency ω_C^c, divides the $\omega_C - \Omega$ plane in two different areas. For a given geometrical confinement frequency Ω and a cyclotron frequency $\omega_C > \omega_C^c$ we can define the usual polaron cyclotron mass m_C^* (12), which is enhanced by the geometrical confinement of the QWW. But for smaller cyclotron frequencies $\omega_C < \omega_C^c$ the radicand of the square root in (12) becomes negative, and so this mass definition would not result in a real polaron cyclotron mass. Hence, in a cyclotron resonance experiment using the optical transition $E_0 \to E_1$ it is only possible to measure the polaron cyclotron mass if $\omega_C > \omega_C^c$. For smaller magnetic fields cyclotron resonance cannot be used to investigate the polaron cyclotron mass.

In figure 5 the value of the critical cyclotron frequency ω_C^c is plotted against the geometrical confinement frequency Ω. To get a deeper insight into what happens for smaller magnetic fields we expand the difference of the energy shifts $\Delta E_1(0) - \Delta E_0(0)$ in second-order RSPT, starting from (14) and (15), in a power series of $\xi = \omega_C/\omega_L$. In the case of

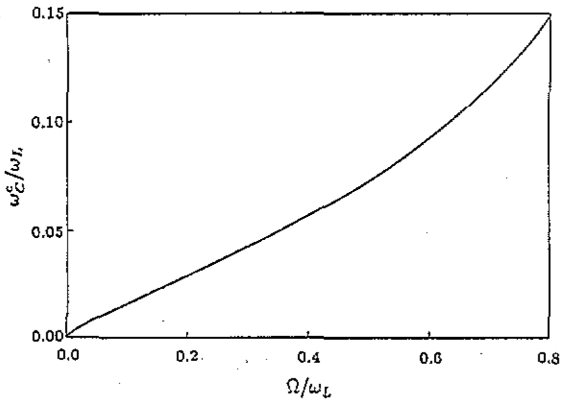

Figure 5. Critical cyclotron frequency ω_C^c of the QID magnetopolaron against confinement frequency Ω of an InSb QWW calculated with IWBPT.

$\xi \to 0$ we get the following results. For vanishing geometrical confinement frequency Ω we obtain

$$\Delta E_0(0) = -\alpha\tfrac{1}{2}\pi\left(1 + \tfrac{1}{8}\xi + \tfrac{1}{128}\xi^2 + \mathcal{O}(\xi^3)\right) \tag{17a}$$

$$\Delta E_1(0) - \Delta E_0(0) = -\alpha\tfrac{1}{8}\pi\left(\xi + \tfrac{9}{8}\xi^2 + \tfrac{145}{128}\xi^3 + \mathcal{O}(\xi^4)\right) \tag{17b}$$

which agrees with the result for the 2D magnetopolaron [3]. But for finite confinement frequency Ω we obtain [12]

$$\Delta E_0(0) = -\alpha\left(a_0 + a_2\xi^2 + \mathcal{O}(\xi^4)\right) \tag{18a}$$

$$\Delta E_1(0) - \Delta E_0(0) = -\alpha\left(b_0 + b_2\xi^2 + \mathcal{O}(\xi^4)\right) \tag{18b}$$

with

$$a_0 = \frac{1}{\sqrt{\pi\eta}}\int_0^\infty \mathrm{d}t\, t^{-1/2}\mathrm{e}^{-t/\eta}\, K\left(\sqrt{\frac{\mathrm{e}^{-t}-1+t}{t}}\right)$$

$$a_2 = \frac{1}{4\sqrt{\pi\eta}\eta^2}\int_0^\infty \mathrm{d}t\, t^{-3/2}\mathrm{e}^{-t/\eta}\left[K\left(\sqrt{\frac{\mathrm{e}^{-t}-1+t}{t}}\right)\times\frac{2-5t+2t^2-(4-6t)\mathrm{e}^{-t}+(2-t)\mathrm{e}^{-2t}}{\mathrm{e}^{-t}-1+t} + K'\left(\sqrt{\frac{\mathrm{e}^{-t}-1+t}{t}}\right)\frac{2-2t-(4-3t+t^2)\mathrm{e}^{-t}+(2-t)\mathrm{e}^{-2t}}{\sqrt{t(\mathrm{e}^{-t}-1+t)}}\right]$$

$$b_0 = \frac{2}{\sqrt{\pi\eta}}\int_0^\infty \mathrm{d}t\, t^{-1}\mathrm{e}^{-t/\eta}\frac{\sinh^2(t/2)}{\sqrt{\mathrm{e}^{-t}-1+t}}K'\left(\sqrt{\frac{\mathrm{e}^{-t}-1+t}{t}}\right)$$

and

$$b_2 = \frac{1}{2\sqrt{\pi\eta}\eta^2}\int_0^\infty \mathrm{d}t\, t^{-2}\mathrm{e}^{-t/\eta}\frac{\sinh^2(t/2)}{\mathrm{e}^{-t}-1+t}\left[K'\left(\sqrt{\frac{\mathrm{e}^{-t}-1+t}{t}}\right)\times\frac{8-t-2t^2+2t^3-(24-18t-t^2-2t^3)\mathrm{e}^{-t}+(24-9t+t^2)\mathrm{e}^{-2t}-8\mathrm{e}^{-3t}}{(1-\mathrm{e}^{-t})(\mathrm{e}^{-t}-1+t)^{1/2}} + t^{-1/2}K''\left(\sqrt{\frac{\mathrm{e}^{-t}-1+t}{t}}\right)\left(2-2t-(4-3t+t^2)\mathrm{e}^{-t}+(2-t)\mathrm{e}^{-2t}\right)\right]$$

where $\eta = \Omega/\omega_L$. If one compares (18) with (17) one can see that the functional structure of both equations differs appreciably. The basic difference is caused by the occurrence of different powers of ξ. In the case of $\Omega \neq 0$ the power series contains only even powers of ξ. For $\omega_C \to 0$ the difference $\Delta E_1 - \Delta E_0$ vanishes for $\Omega = 0$ but remains finite for $\Omega \neq 0$. It is obvious that in the limit of small magnetic fields ($B \to 0$) the polaron cyclotron mass can only result in the polaron mass if the electron–phonon contribution to the energy splitting $\Delta E_1 - \Delta E_0$ vanishes. But, due to the quantum confinement, this is not the case in the Q1D system. Hence, it is impossible to measure the polaron mass ($B \to 0$) in QWW using cyclotron resonance. Nevertheless, it is straightforward but tedious to show that (18) approaches (17) including all powers of ξ for $\Omega \to 0$. In the weak magnetic field limit the polaron cyclotron mass of (12), using (18), is given by

$$\frac{m_C^*}{m_e} = \frac{\xi}{\sqrt{-\alpha b_0(2\eta - \alpha b_0)}}. \tag{19}$$

Under the condition $\alpha b_0 < 2\eta$, which is valid for small but finite confinement frequencies Ω, one can see that this definition of the polaron cyclotron mass does not give a real value.

4. Conclusions

In conclusion, we have calculated the polaron cyclotron mass of Q1D magnetopolarons in QWW. Our results are valid for zero temperature and arbitrary magnetic field strength. It is shown that the polaron cyclotron mass increases for $B \to 0$ because of the electrostatic confinement, different to this mass of the 3D and Q2D magnetopolaron in perpendicular magnetic fields. Level crossing and following the existence of a resonant magnetopolaron arises only under the condition $\omega_L > N\Omega$ and at this magnetic field where $\omega_C = [(\omega_L/N)^2 - \Omega^2]^{1/2}$ is valid. Merkt and Sikorski [8] have shown the possibility of producing InSb MOS QWW with subband separation energies up to $\hbar\Omega = 9$ meV. For QWW with those and higher subband separation energies it should be possible to observe in experiment the polaronic frequency shift to lower values due to the resonant magnetopolaron effect, if one extends the cyclotron resonance measurement to such a magnetic field where the level crossing occurs. Further, the unusual increase of the polaron cyclotron mass for lower magnetic fields should also be measurable.

To improve on these results one has to include in the calculation the non-parabolicity of the conduction band (the band structure effect), the non-parabolicity of the confinement potential, the finite width of the QWW in the growth direction and, if many electrons are present, occupation and screening effects.

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